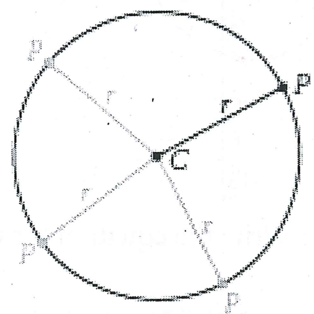
8.1/8.2 ~ Applying the Distance Formula and Circles

Objectives:

1. Apply the distance formula to problems to find a locus of points.
2. Write the equation of a circle in standard form.
3. Write the equation of a circle from a graph.
4. Graph a circle from its equation.

Definition of a Circle

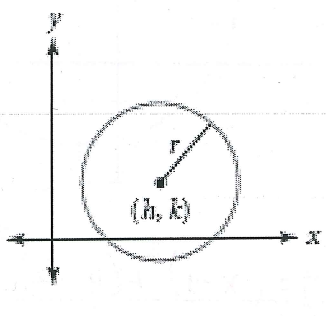
A **circle** is a locus of points P in a plane, that are a constant distance, r, from a fixed point, C. Symbolically, $PC = r$. The fixed point is called the **center** and the constant distance is called the **radius**.



Equation of a Circle

The **standard form** of the equation of a circle with center (h, k) and radius r is

$$(x - h)^2 + (y - k)^2 = r^2$$



Example 1: Identify the center and the radius from the circle equations below:

a. $(x+3)^2 + (y+1)^2 = 64$

$(-3,-1)$

$r=8$

b. $(x-5)^2 + (y-15)^2 = 100$

$(5,15)$

$r=10$

c. $(x+12)^2 + (y-9)^2 = 55$

$(-12,9)$

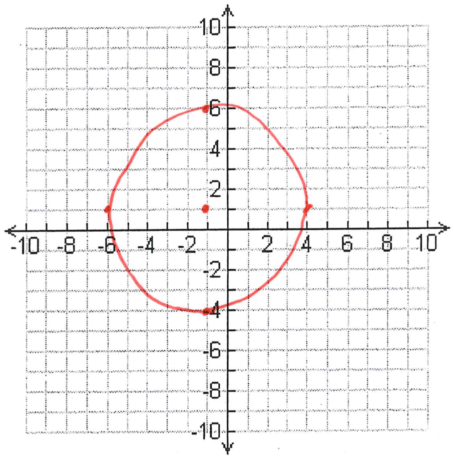
$r=\sqrt{55}$

$r\approx 7.416$

Example 2: Graph the circle represented in the equations below:

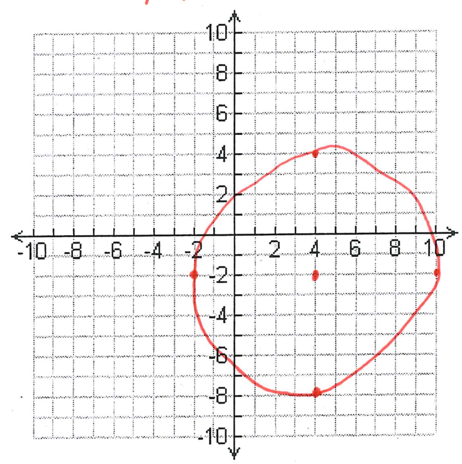
a. $(x+1)^2+(y-1)^2=25$

$(-1,1)$ $r=5$

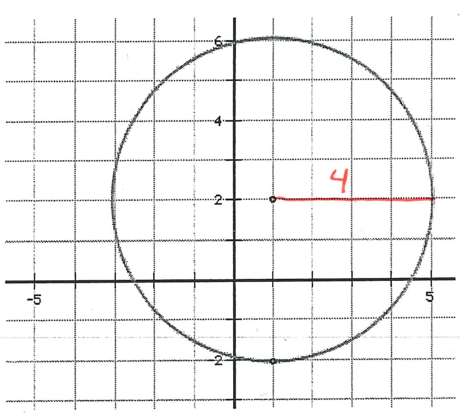


b. $(x-4)^2+(y+2)^2=36$

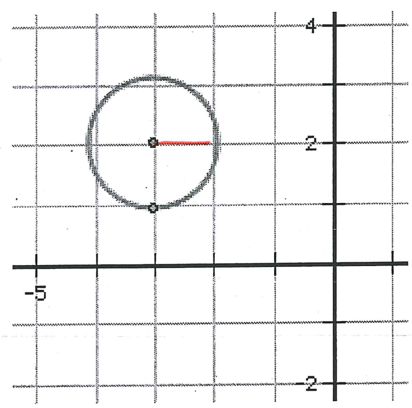
$(4,-2)$ $r=6$



Example 3: Write the equation for the circles graphed below:



$(x-1)^2+(y-2)^2=16$



$(x+3)^2+(y-2)^2=1$

Distance Formula

The distance, d, between two points on a coordinate plane, $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, is given by the formula

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Example 1: Find the distance between the points (9, 4) and (6, 1).

$d=\sqrt{(6-9)^2+(1-4)^2}$

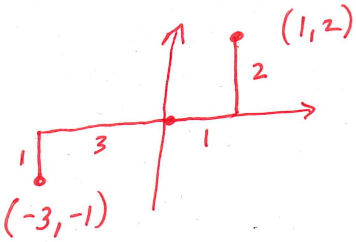
$d=\sqrt{(-3)^2+(-3)^2}$

$d=\sqrt{9+9}$

$d=\sqrt{18}$

$d\approx 4.243$

Example 2: Hayley and Katie both start from the same spot. Hayley walks 1 mile east and 2 miles north. Katie walks 3 miles west and 1 mile south. How far apart are Hayley and Katie?



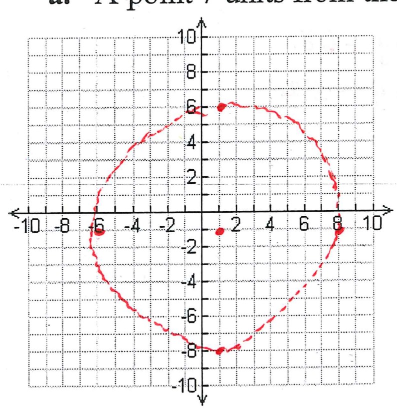
$$d = \sqrt{(1--3)^2 + (2--1)^2}$$
$$= \sqrt{4^2 + 3^2}$$
$$= \sqrt{16 + 9}$$
$$= \sqrt{25}$$
$$d = 5 \text{ miles}$$

Locus (plural ~ Loci):

A set of points that meet a given condition.

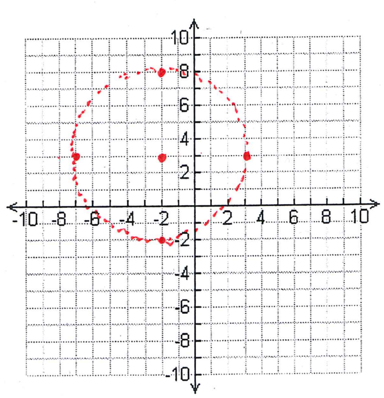
Example 3: Sketch the loci of points and write an equation to represent the locus of points described:

a. A point 7 units from the point (1, -1).



$$d = \sqrt{(x-x_1)^2 + (y-y_1)^2}$$
$$7 = \sqrt{(x-1)^2 + (y+1)^2} \quad \leftarrow \text{Square Each Side}$$
$$49 = (x-1)^2 + (y+1)^2$$

b. A point 5 units from the point (-2,3).



$$5 = \sqrt{(x+2)^2 + (y-3)^2}$$
$$25 = (x+2)^2 + (y-3)^2$$

Example 4: Find the equation of the locus of points that are equidistant from the point (1,3) and (5,6).

$$\left(\sqrt{(x-1)^2+(y-3)^2}\right)^2=\left(\sqrt{(x-5)^2+(y-6)^2}\right)^2$$

$$(x-1)^2+(y-3)^2=(x-5)^2+(y-6)^2$$

$$x^2-2x+1+y^2-6y+9=x^2-10x+25+y^2-12y+36$$

$$x^2-2x+y^2-6y+10=x^2-10x+y^2-12y+61$$

$$-2x-6y-10=-10x-12y+61$$

$$+10x+12y \quad +10 \qquad +10x+12y \quad +10$$

$$\boxed{8x+6y=71}$$

Example 5: Find the equation of the locus of points that are equidistant from the points (-4,6) and (2, -8).

$$\sqrt{(x+4)^2+(y-6)^2}=\sqrt{(x-2)^2+(y+8)^2}$$

$$x^2+8x+16+y^2-12y+36=x^2-4x+4+y^2+16y+64$$

$$x^2+8x+y^2-12y+52=x^2-4x+y^2+16y+68$$

$$-x^2+4x-y^2-16y-52 \qquad -x^2+4x-y^2-16y-52$$

$$\boxed{12x-28y=16}$$